

Applications and Rewriting of Omnigraphs – Exemplified in the Domain of MDD

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Abstract. Graph rewrite systems provide only elementary primitives – many applications require more complex structures though. We present a rewrite system for omnigraphs, a formal extension of hypergraphs with the ability to connect multiple nodes *and edges* with a single edge. We exemplify the adequacy of this approach in the domain of Model Driven Development (MDD): Using our system trivializes the representation and transformation of advanced UML structures most modeling environments dodge today.

Key words: graph rewriting, hypergraph, omnigraph, supergraph

1 Introduction

Graph rewrite systems elegantly handle various tasks; they have sound and concise fundamentals and their computational power is Turing equivalent. But the operational primitives of current graph rewrite systems are quite elementary, quite assembler-language-like. Several application domains demand more powerful primitives. One example for such a domain is the representation and transformation of UML within MDD.

UML class diagrams allow n -ary associations which are de facto hyperedges [1] (cf. Fig. 2 for example). Furthermore, they allow relations between associations (cf. Fig. 3). In order to express these relations directly, we would need to additionally allow *edges* to be end points of edges – and that is precisely what omnigraphs¹ are about.

In 1998, Minas showed the advantage of hypergraphs over traditional graphs for representing various kinds of diagrams [9], but no available graph rewrite system has support for hypergraphs so far, not to mention omnigraphs. Therefore, we developed languages for model definition, graph definition and rewrite specifications for omnigraphs. Compilers [18] translate these languages into semantically equivalent definitions for a traditional graph rewrite system. In this paper, we present these languages (Section 2), their theoretical fundamentals (Section 3), and the functionality of the compilers (Section 4).

¹ In previous work, we referred to ‘omnigraphs’ as ‘supergraphs’, but we changed the name in order to avoid further confusion with the antonym of ‘subgraphs’.

2 Omnigraphs in Use – A Problem-oriented Introduction

Before giving a formal definition of omnigraphs in the next section, we will introduce omnigraphs by means of their application to a specific problem: representation and transformation of UML. We thereby demonstrate how the concepts of omnigraphs ease the handling of advanced UML structures.

2.1 UML Models as Omnigraphs

We show how to represent UML class diagrams using the syntax of our custom-made graph rewrite system *Ogre* (OmniGraph REwriting²). For the complete syntax of *Ogre*, please refer to [18]. We have taken all examples from the UML Superstructure Specification version 2.1.1 [12].



Fig. 1: Simple association, Fig. 7.19 from the UML Superstructure Specification

Defining a Model. Figure 1 shows a simple association between two classes *A* and *B* with multiplicities and the roles *endA* and *endB*. Listing 1 shows the definition of an accordant model for omnigraphs: *Ogre* provides nodes, omniedges, and roles as graph primitives. A definition starts with the type of the primitive, so line 1 defines a type for nodes named *Class* and line 4 defines a type for omniedges named *Association*. Constraints for omniedges (in parentheses) specify the allowed types of roles. The constraint in line 4 states that the omniedges need a least one (+) end point of role type *AssociationEnd*. In line 5, this class of roles is defined with the constraint that it is only applicable on nodes of the type *Class*. An *AssociationEnd* has two attributes: *name* and *multiplicity*. Please note that already managing these two attributes is cumbersome in graph rewrite systems that do not support attributed endpoints on edges.

```

1 node Class {
2   name : string;
3 }
4 omniedge Association (AssociationEnd+);
5 role AssociationEnd (Class) {
6   name : string;
7   multiplicity : string;
8 }

```

Listing 1: Model for UML classes and associations

² Previous name: SUGR – SUperGraph Rewriting

Instantiation. Having defined the model, we will now set up an instance of this model – an omnigraph representing the UML class diagram depicted in Figure 1. Let’s begin with the one line statement depicted in Listing 2: The graph definition starts with an omniedge of the type *Association*. Inside the body of the omniedge we define two new nodes of type *Class*. Their corresponding roles – *AssociationEnd* in both cases – follow, separated by vertical bars. The two nodes with their roles form the end points of the surrounding omniedge. In general, nodes and omniedges are declared by an optional identifier (before) and a type (after the colon). The declaration of an omniedge additionally has a body (in square brackets) defining its end points. As we will not refer to any of the declared graph elements again in the code snippet in Listing 2, we omit the identifiers here. But what we effectively lack in this declaration are attributes!

```
1 : Association[: Class| AssociationEnd : Class| AssociationEnd]
```

Listing 2: Graph definition for a simple association

Now, we define a new graph *with* attributes (cf. Listing 3). This time, we also show how to declare identifiers and how to reference graph elements. In line 1 and 2, the required class nodes are defined. They have identifiers (*a* and *b*) and attributes to hold the *names* “A” and “B”. In line 4 and 5, *a* and *b* are referenced (indicated by the @ character) and used to define end points. The nodes take the role *AssociationEnd* as above, but this time we additionally define attributes for the end points: “endA” respectively “endB” as value for the *name* attribute, and “*” as value for the *multiplicity* attribute of the respective ends of the association. We now have a true representation of the association from Figure 1.

```
1 a: Class(%name="A")
2 b: Class(%name="B")
3 : Association[
4   @a| AssociationEnd(%{ name="endA", multiplicity="*"})
5   @b| AssociationEnd(%{ name="endB", multiplicity="*"})
6 ]
```

Listing 3: Graph definition for a simple association (with attributes)

Ternary Associations. UML enables the declaration of *n*-ary associations which are not directly expressible by simple binary associations. The ternary association in Figure 2 is an example.

As a binary edge is only a special case of an omniedge we do not need to extend the previous model from Listing 1: It already accepts an arbitrary number of *AssociationEnds* for each *Association*. So we can immediately denote the example as an omnigraph definition as shown in Listing 4. We simply define a third end point inside the *Association* omniedge. The *Class* nodes are again defined within the omniedge body.

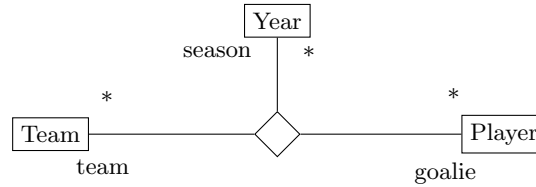


Fig. 2: Ternary association, Fig. 7.21 from the UML Superstructure Specification

```

1 : Association[
2   : Class(%name="Team") | AssociationEnd(%{name="team", multiplicity="*"})
3   : Class(%name="Year") | AssociationEnd(%{name="season", multiplicity="*"})
4   : Class(%name="Player") | AssociationEnd(%{name="goalie", multiplicity="*"})
5 ]

```

Listing 4: Graph definition for the ternary association

Higher Order Predicates. Figure 3 shows an example for a second order predicate in UML class diagrams: The $\{xor\}$ constraint is a predicate over two associations which are predicates over (*Class*-) nodes themselves. Omnigraphs remove the restriction of hypergraphs by allowing higher order predicates. To express this UML constraint, our model needs an extension: a new type of omniedge named *Constraint* and a new role *ConstraintEnd*. We only want omniedges of the type *Association* to take this role, so we restrict the role to this type. The complete model definition is shown in Listing 5. The graph definition in Listing 6 consist of two associations (line 1 and 4). The constraint is an additional omniedge with the two associations as end points (line 7). The two associations share a common node, so this node is identified by a (line 2) and referenced (line 5).

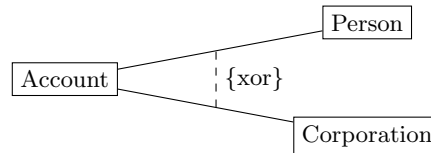


Fig. 3: $\{xor\}$ constraint, Fig. 7.34 from the UML Superstructure Specification

2.2 Transforming UML with Ogre

After having shown how to represent a UML class diagram as omnigraph, we will show how to define an elementary transformation: We now want to transform the ternary association form the preceding example (cf. Figure 2 and Listing 4) into adequate binary associations. This transformation is an inevitable step in every model driven process; we repeat it every time we decompose our model to obtain executable code. The rule according to this transformation demonstrates the pragmatical simplification of writing rules the omnigraph approach initially

```

1 node Class {
2   name : string;
3 }
4 omniedge Association (AssociationEnd+);
5 role AssociationEnd (Class) {
6   name : string;
7   multiplicity : string;
8 }
9 role ConstraintEnd (Association);
10 omniedge Constraint (ConstraintEnd+) {
11   type : string;
12 }

```

Listing 5: Model for UML classes, associations, and constraints

```

1 a1: Association[
2   a: Class(%name="Account") | AssociationEnd
3   : Class(%name="Person") | AssociationEnd]
4 a2: Association[
5   @a | AssociationEnd
6   : Class(%name="Corporation") | AssociationEnd]
7 : Constraint[@a1 | ConstraintEnd @a2 | ConstraintEnd] (%type="xor")

```

Listing 6: Graph definition for the {xor} constraint

has been designed for: no need to think about any extra nodes and edges, their names, types, and directions – simply because we have omniedges with attributed end points.

Transforming Ternary Associations. The rewriting rule in Listing 7 decomposes ternary associations into adequate binary associations. The left-hand side, the **pattern** graph, matches an omniedge *a* of type *Association* with three end points *c1*, *c2*, and *c3* of type *Class* and with roles *ae1*, *ae2*, and *ae3* of type *AssociationEnd*. The syntax is similar to graph definitions: we use an identifier followed by a colon and a type. The role type is separated by a vertical bar; in contrast to graph definitions we can use identifiers for roles. On the right-hand side of the rule, the **modify** graph, we first delete the ternary association *a* (line 9) and create a new node *c4* of type *Class* (line 10) – serving as new connection node. Then we create three new associations between the connection node and the former end points of the ternary association. Finally, we have to set the attributes for the new graph elements, which is done in the **eval** section. Line 15, for example, creates the name for the newly created class *c4*: It consist of the names of the classes *c1*, *c2*, and *c3* and the suffix “Triple”.

Figure 4 shows the ternary association after transforming it into an extra class and appropriate binary associations. The rule also changed all multiplicities as necessary for a correct transformation.

The Rewriting Semantics of OGRE. In OGRE, rules consist of a **pattern** graph and a **replace** or a **modify** graph. Each element of these graphs has a name, either user defined or internally defined. Consider a graph element defined in the **pattern** part: If its name is used in the **replace** graph, the denoted

```

1  pattern {
2    a: Association[
3      c1: Class| ae1: AssociationEnd
4      c2: Class| ae2: AssociationEnd
5      c3: Class| ae3: AssociationEnd
6    ];
7  }
8  modify {
9    delete(a);
10   c4: Class;
11   : Association[c4| ae11: AssociationEnd c1| ae21: AssociationEnd];
12   : Association[c4| ae12: AssociationEnd c2| ae22: AssociationEnd];
13   : Association[c4| ae13: AssociationEnd c3| ae23: AssociationEnd];
14   eval {
15     c4.name = c1.name+c2.name+c3.name+"Triple";
16     ae11.multiplicity = ae1.multiplicity; ae11.name = ae1.name;
17     ae21.multiplicity = "1";
18     ae12.multiplicity = ae2.multiplicity; ae12.name = ae2.name;
19     ae22.multiplicity = "1";
20     ae13.multiplicity = ae3.multiplicity; ae13.name = ae3.name;
21     ae23.multiplicity = "1";
22   }
23 }

```

Listing 7: Rule for processing ternary associations

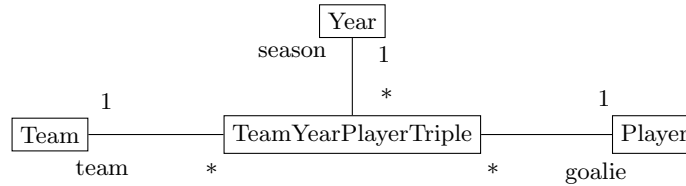


Fig. 4: Ternary Association after the transformation by the rule

graph element will be kept during the execution of the rule. Otherwise the graph element will be deleted from the host graph. A graph element is created in the host graph by defining a name in the **replace** graph. Anonymous graph elements in a **replace** graph always create new elements in the host graph. Using a name multiple times has the same effect as a single occurrence.

The **modify** variant is syntactic sugar for copying the **pattern** graph to the **replace** graph – in this case deletions from this replacement graph are triggered by the **delete** keyword; additions work the usual way. In case of a conflict between deletion and preservation, deletion is prioritized. It is convenient to use the **modify** variant for modifying only small parts of a large pattern graph.

For a proper graph rewriting system, we need a sound approach on how incident objects are treated when other objects are deleted. Traditional graph rewrite systems can be classified according to SPO or DPO, but both approaches are obviously not applicable for omnigraphs. Our definition of omnigraphs (cf. Section 3.2) allows edges to have an arbitrary number of end points, including zero. Deletion of an incident node of an omniedge just reduces the number of end points of that omniedge. Deletion of an omniedge always requires an explicit

statement, and removing it does not bother the objects it connected any further. Thus deletion can never lead to an invalid omnigraph.

2.3 Advanced UML Structures in Practice

The UML structures we are referring to in this publication are surprisingly uncommon and many software engineers are unfamiliar with these features of UML. Nevertheless, one may have a hard time trying to encode their semantics *without* these structures. In our opinion, this already justifies their existence and their use – leading to the necessity for their support in modelling tools. Some more structures of UML that lend themselves to be realized via omnigraphs are attributed associations, qualified associations, fork-, join, merge- and decision nodes, or duration constraints.

3 Formal Definition of Omnigraphs

Before we present the formalism we discuss some issues regarding our approach of generalizing “direction”. This discussion should explain our perspective on hypergraphs and demonstrate that the given definition is adequate.

3.1 Roles

In a traditional (directed) graph, each edge has a direction, a point of origin and an aiming point. But how can we specify something comparable for omnigraphs with arbitrary numbers of end points? The following paragraphs presents an approach that renders the ordinary directed edge a special case of a more general concept.

Every formalism – every way of representing information – provides certain primitives to store pieces of information and other primitives to relate these fragments. The available primitives determine the semantics that can be encoded directly with this formalism. Graphs, too, are just a special way of representing information. Their primitives are usually nodes and edges with labels. We use the labels to store pieces of information. Contiguity relates the information stored in a graph.

On closer inspection, one can see that there is a third primitive in graphs that allows information storage: the direction of edges. Direction enables us to store several extra bits per edge – one extra bit if only unidirectional edges are permitted, two extra bits if multidirectional edges are permitted, and no extra bits if only undirected edges are permitted in the graph. Initially, these bits encode the direction of an edge. Additional information, for example “who loves whom” or “which code block precedes another”, is an interpretation that has been agreed on. This agreement constitutes which bit-value represents which role in the relation. Thus we are effectively interested in the *roles* an edge assigns and *not* in its direction.

Taking into account that we are interested in roles rather than direction, we could as well provide roles immediately in our way of representing information. Instead of storing one direction per edge, each end of an edge is assigned a role. The advantage of this approach is that it scales a lot better: Now, it is irrelevant how many ends (these “ends” are called “tentacles” in hypergraphs) an edge has, including the cases “one” and “zero”.

Another conclusion we can draw from the above consideration is that the roles we are effectively interested in are seldom “source” and “target”. We would rather allow arbitrary roles. Accordingly, the number of available roles does not need to be limited to two.

For these reasons, our omniedges do not have a direction and no inherent order or numerical limitations of their tentacles. Instead, each tentacle is assigned a role out of an arbitrary, finite set of roles. Initially, the combinations of roles within one omniedge are unrestricted. One might want to impose constraints about the legal role sets per omniedge, though.

A classic approach to assign meaning to the tentacles of a hyperedge (or the components of a tuple) is position: Any term-based syntax for the declaration of hyperedges imposes a sequence of tentacle declarations. One can use the position within this sequence to assign meaning to a tentacle. The downside of this approach is that the sequence always needs to be specified completely and in order, and that no meaning can be assigned to additional tentacles. In programming languages like Eiffel or Visual Basic, some of these drawbacks can be resolved by *named* function arguments. The role-based approach is a generalization thereof.

For illustration, we show that an ordinary (directed, two-ended) edge can be expressed immediately in terms of this approach: It is an omniedge with two tentacles of which one has the role “source” and the other has the role “target”. A traditional graph is thus completely representable as omnigraph. As also omnigraphs are representable via traditional graphs (cf. Section 4) both formalisms are *theoretically* equally expressive. Yet the *practical* expressiveness of omnigraphs is more suitable for certain applications as we show in this publication.

3.2 Definition

Definition 1 (Omnigraph). *Let V, O, T, R be arbitrary finite pairwise disjoint³ sets, $C := V \cup O$, and src , tgt , and rol total but not necessarily injective or surjective functions with*

$$\begin{aligned} src &: T \longrightarrow O \\ tgt &: T \longrightarrow C \\ rol &: T \longrightarrow R \end{aligned}$$

then the 7-tuple $G = (V, O, T, src, tgt, R, rol)$ is an omnigraph.

³ For clearness we require V, O, T, R to be pairwise disjoint. Formally it is only necessary, that $V \cap O = \emptyset$ holds.

Explanation and Implications. We call V the vertices, O the omniedges, T the tentacles, and R the roles of an omnigraph. $C = V \cup O$ is the set of all connectible objects. The tentacles link the elements of C to their connections $o \in O$; these links are denoted by the `src` and `tgt` functions assigning the obvious direction⁴ to the tentacle. It is specific to omnigraphs that the tentacles can only start at omniedges but end at omniedges and vertices. So by definition, no vertex can have an outgoing tentacle whereas incoming tentacles are allowed. Each tentacle linking a connectible object $c \in C$ to an omniedge $o \in O$ also specifies a role $r \in R$ that c takes in o . This is denoted by the `rol` function.

As omniedges may connect other omniedges, the tentacles are directed to make clear which omniedge establishes the connection between the other ones. It is only usual to utilize the concept of ‘direction’ here – but it is not necessary: It is sufficient to *somehow* distinguish which tentacles belongs to which omniedge. Accordingly, it is irrelevant which direction the tentacles exactly have, as long as it is consistent for all omniedges.

Multigraphs are defined by their allowance for multiple edges between two distinct nodes. For omnigraphs this property is obtained by our function-based definition instead of the commonly used tuple-based definition. Thus an arbitrary number of omniedges can occur between each set $C' \subseteq C$. Furthermore, this property allows multiple tentacles of the same or different roles between an omniedge and one connected object $c \in C$. Moreover, if not every node or omniedge has an incident tentacle `src` and `tgt` are not surjective. We require `src` and `tgt` to be *total* such that no dangling tentacles can occur.

Omnigraphs as defined here have two properties that may appear strange, but are harmless consequences of the generality of the concept: (a) Omniedges may have one or zero tentacles and (b) omniedges may connect to themselves.

Remark. Our definition is rather close to the function-based definition of ordinary hypergraphs. One *could* picture the set T as “edges” and C as “vertices”. But this picture is only half true, because the set C has an internal structure: The “edges” can only start at elements $o \in O \subseteq C$ and end at *any* element $c \in C = O \cup V$. This way, the property of being representable as bipartite graphs (like ordinary hypergraphs) is lost.

Formally speaking, our definition is a direct extension of hypergraphs. But we define names and interpretations of the sets C , O , V , and T to suit our need for a vocabulary of concepts on a higher level of abstraction.

3.3 Examples

We will present two of the examples from Section 2 as formal omnigraph definitions. The first example is the formal definition of the hyperedge depicted in Figure 2. The second example is the formal definition of the constraint edge between two association edges depicted in Figure 3.

⁴ Please note that only the tentacles but not the omniedges themselves are directed.

The ternary association from Figure 2 has three vertices: *Team*, *Year*, and *Player*, and an omniedge *Association*. The omniedge has three tentacles t_0 , t_1 , and t_2 with the roles *team*, *season*, and *goalie*. In formulae, this looks like:

$$\begin{aligned}
V &= \{Team, Year, Player\} \\
O &= \{Association\} \\
T &= \{t_0, t_1, t_2\} \\
R &= \{team, season, goalie\} \\
src(t_0) &= Association & tgt(t_0) &= Team & rol(t_0) &= team \\
src(t_1) &= Association & tgt(t_1) &= Year & rol(t_1) &= season \\
src(t_2) &= Association & tgt(t_2) &= Player & rol(t_2) &= goalie
\end{aligned}$$

The formal definition of the {xor} constraint has three vertices: *Account*, *Person*, and *Corporation*, and three omniedges: *Association₀*, *Association₁*, and *Constraint*. Each omniedge has two tentacles, so that we have in total six tentacles, but only two different roles *associationEnd* and *constraintEnd*. The visualisation of the formal definition is shown in Figure 5.

$$\begin{aligned}
V &= \{Account, Person, Corporation\} \\
O &= \{Association_0, Association_1, Constraint\} \\
T &= \{t_{a0}, t_{a1}, t_{a2}, t_{a3}, t_{c0}, t_{c1}\} \\
R &= \{associationEnd, constraintEnd\} \\
src(t_{a0}) &= Association_0 & tgt(t_{a0}) &= Account & rol(t_{a0}) &= associationEnd \\
src(t_{a1}) &= Association_0 & tgt(t_{a1}) &= Person & rol(t_{a1}) &= associationEnd \\
src(t_{a2}) &= Association_1 & tgt(t_{a2}) &= Account & rol(t_{a2}) &= associationEnd \\
src(t_{a3}) &= Association_1 & tgt(t_{a3}) &= Corporation & rol(t_{a3}) &= associationEnd \\
src(t_{c0}) &= Constraint & tgt(t_{c0}) &= Association_0 & rol(t_{c0}) &= constraintEnd \\
src(t_{c1}) &= Constraint & tgt(t_{c1}) &= Association_1 & rol(t_{c1}) &= constraintEnd
\end{aligned}$$

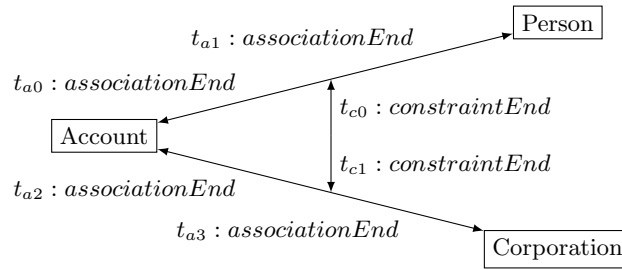


Fig. 5: Formal graph for the {xor} constraint (names of omniedges omitted).

3.4 Extensions

For an efficient usage of a graph rewriting *tool* we need (a) labels on nodes, edges, and roles, and want (b) these labels to obey certain constraints, i.e. typing. Furthermore, (c) inheritance relations among these types are needed for an easier declaration of rules. Accordant extensions to the formal basis of omnigraphs can be defined the usual way without difficulty.

Ogre implements these features. It seamlessly incorporates them from the underlying graph rewrite system (cf. Section 4). In contradiction to some formalizations of hypergraphs, we assign the type of an omniedge directly to it and do not derive it from the number or types of tentacles. In particular, omniedges of a certain type may have arbitrary numbers of tentacles. However, Ogre supports constraints about the legal role types for each omniedge type.

4 Implementation

To avoid developing a graph rewrite system from scratch, we chose to decompose omnigraph model-, rule-, and graph-definitions to model-, rule-, and graph-definitions for the traditional graph rewrite system GrGen.NET [6]. Ogre provides three compilers for this task. As the space in this paper is limited, we only give a rough outline of these transformations. Details can be found in [5], the compilers including source code are available from [18].

4.1 Mapping Ogre Definitions to GrGen.NET Definitions

As GrGen.NET has no support for omniedges and roles, we need a mapping to translate omniedges and roles into nodes and edges, the primitives provided by GrGen.NET. We map omniedges by introducing an additional interconnection node. As a consequence each tentacle becomes an independent edge between the interconnection node and the node connected by the tentacle; the role of the tentacle is mapped to the type of the according edge. This approach is quite obvious and well-known from treating hyperedges as bipartite graphs. But in the context of omniedges we have to pay special attention to the direction of decomposed edges: their tentacles are directed, as discussed in Section 3.2. Correspondingly, we realize tentacles with directed edges. Figure 6 shows the mapping for the {xor} constraint example from Figure 3.

Mapping Models. Listing 8 shows the result of mapping our model from Listing 1 to GrGen.NET syntax. We can see that the omniedge type *Association* has become a node type (line 9) and the role type *AssociationEnd* has become an edge type (line 4). Line 5 shows a GrGen.NET constraint defining the allowed source and target types for this edge type.

Figure 7 shows the visualization of the mapped model for the ternary association. We can clearly see the interconnection node with the omniedge type and the edges with the role types.

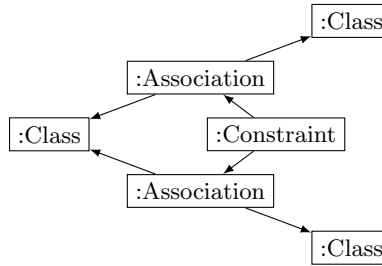


Fig. 6: Mapping of the {xor} constraint

```

1 node class Class extends NODE {
2   name : string;
3 }
4 edge class AssociationEnd extends ROLE
5   connect Association[1:.*] -> Class[*] {
6     name: string;
7     multiplicity: string;
8   }
9 node class Association extends OMNIEDGE;

```

Listing 8: Model from Listing 1 after translation in *GrGen.NET* syntax

Mapping Graphs. Listing 9 shows the ternary association from Listing 4 after the mapping to GrGen.NET syntax. The omniedge *Association* is decomposed into an interconnection node (line 1) and edges representing the tentacles (line 3, 5, and 7). In GrGen.NET edges are denoted by an arrow from the source to the target node. Identifier, type and attributes of the edge are stated between beginning and end of that arrow. The dollar sign is a build-in attribute of GrGen.NET keeping the identifier of the graph primitive for debugging purposes. The *Class* nodes and their declared attributes are preserved during the mapping.

This listing shows how the concept of omnigraphs disburdens the user from the necessity to fragment his or her thoughts for the input into a traditional graph rewrite system. Besides this semantic advantage, the OGRE syntax eliminates the need to constantly repeat identifiers for miscellaneous nodes and edges. This is not so much a quantitative but a qualitative alleviation, as repeating numerous – only technically induced – identifiers is an error prone work.

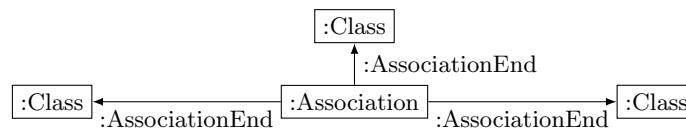


Fig. 7: Mapping of the **model** for the ternary association

```

1 new s_3: Association($="s_3")
2 new n_0: Class($="n_0", name="Team")
3 new s_3 -: AssociationEnd(name="team", multiplicity="*")-> n_0
4 new n_1: Class($="n_1", name="Year")
5 new s_3 -: AssociationEnd(name="season", multiplicity="*")-> n_1
6 new n_2: Class($="n_2", name="Player")
7 new s_3 -: AssociationEnd(name="goalie", multiplicity="*")-> n_2

```

Listing 9: Graph from Listing 4 after translation in *GrGen.NET syntax*

Mapping Rules. Basically rules are processed by treating the left-hand and right-hand side patterns individually as graph definitions and mapping them separately. As the syntax for patterns is quite similar to graph definitions we reuse the mappings for graph definitions with some minor changes. Listing 10 shows the rule from Listing 7 after mapping to GrGen.NET syntax. The content of the `eval` section can just be copied as it is, because it does not need to be changed by the mapping.

```

1 pattern {
2   a: Association;
3   a -ae1: AssociationEnd-> c1: Class;
4   a -ae2: AssociationEnd-> c2: Class;
5   a -ae3: AssociationEnd-> c3: Class;
6 }
7 modify {
8   delete(a);
9   c4: Class;
10  x2: Association;
11  x2 -ae11: AssociationEnd-> c4;
12  x2 -ae21: AssociationEnd-> c1;
13  x5: Association;
14  x5 -ae12: AssociationEnd-> c4;
15  x5 -ae22: AssociationEnd-> c2;
16  x8: Association;
17  x8 -ae13: AssociationEnd-> c4;
18  x8 -ae23: AssociationEnd-> c3;
19  eval {
20    c4.name = c1.name+c2.name+c3.name+"Triple";
21    ae11.multiplicity = ae1.multiplicity; ae11.name = ae1.name;
22    ae21.multiplicity = "1";
23    ae12.multiplicity = ae2.multiplicity; ae12.name = ae2.name;
24    ae22.multiplicity = "1";
25    ae13.multiplicity = ae3.multiplicity; ae13.name = ae3.name;
26    ae23.multiplicity = "1";
27  }
28 }

```

Listing 10: Rule from Listing 7 after translation in *GrGen.NET syntax*

Obviously, the semantics of our rules are derived directly from the semantics of GrGen.NET rules [2]. GrGen.NET implements closely the SPO semantics, so deleting an incident node of an edge will also delete that edge. This behaviour is well suited for our needs: If we want to delete an omniedge, we can just delete the interconnection node. The SPO semantics will lead to the deletion of edges representing its tentacles. As we allow omniedges to have an arbitrary number of tentacles (including zero), deleting an incident connectible object (and thus ‘loosing’ a tentacle) cannot lead to an invalid omnigraph (cf. Section 2.2).

5 Related Work

Graph rewrite systems have been under research for several decades. Research in Model Driven Development has lead to a strong demand for model transformation technology. This has brought graph rewriting – as one possible solution for model transformation – to industrial relevance.

In this paper, we discuss omnigraph rewriting. To the best of our knowledge, no system with this capability has been published, yet. In OGRE, omniedges – and thus also hyperedges – are first-class citizens among the graph primitives: They are quasi materialized in the graph and can seamlessly be used in search and replacement patterns. Thus OGRE is also a hypergraph rewriting system. But even for hypergraph rewriting⁵, there is no comparable system available. A property – besides allowing edges to be endpoints of edges – that distinguishes our notion of omnigraph rewriting from the usual *theoretical* notion of hypergraph rewriting is that omniedges have no fixed number of tentacles defining their type. Furthermore, the tentacles of omniedges have no inherent order. Both properties are by design, as we discuss in Section 3.

The Graph eXchange Language (GXL) supports omnigraphs using `rel`- and `rele`-elements. But as already suggested by its name, GXL is not a rewrite system but an exchange format and only serves to store graphs – it can neither store rules nor rewrite sequences. For solely exchanging omnigraphs, GXL would suit very well, but we could not find a tool to exchange omnigraphs graphs with: Holt [8] gives an overview of GXL capable tools, namely GRAS [15], DiaGen [10], Fujaba [13], GenSet [14] and PROGRES [19]. Except DiaGen⁶, all these systems ignore `rel` tags, they have no support for hyper- or omniedges. The same holds true for GROOVE [17].

AGG-graphs [3], a variant of ALR-graphs and the formal basis of AGG, explicitly enable edges between edges. But these edges are only binary, hyperedges are not first-class citizens in AGG-graphs. Instead, AGG-graphs come with a direct support for ‘abstractions’ which in turn are not first-class citizens in omnigraphs. So AGG-graphs and omnigraphs are skew to each other. However, AGG (the tool) has no support for edges between edges.

GReAT [16] and VIATRA [20] partially support edges between edges: GReAT can define edges between edges in models, but has no possibility to use them in graphs or rules. VIATRA can use edges between edges in graphs and rules, but cannot define them in models. In both systems, edges are always binary.

Apart from the support for omniedges, we regard OGRE as ordinary general purpose graph rewrite system. Table 1 compares the features of OGRE with those of Fujaba, GReAT, VIATRA and GrGen.NET [6], some of the most popular general purpose graph rewriting tools today. The criteria are adopted from Czarnecki and Helsen [4].

⁵ Please note that ‘hypergraph rewriting’ and ‘hyperedge replacement’ [11] are different things: In hyperedge replacement, hyperedges are only special left-hand side patterns of replacement rules. Our conception of hypergraph rewriting is much broader.

⁶ DiaGen is a tool for generating diagram editors based on hypergraphs. We aim to develop a general purpose graph rewrite system.

Table 1: Overview Graph Rewriting Tools

	Fujaba	GReAT	VIATRA	GrGen.NET	Ogre
Typed Domain	yes	yes	yes	yes	yes
Type Inheritance	single	multi	multi	multi	multi
Node Attributes	Java types	simple types, enumerations	simple and complex types	simple types, enumerations	simple types
Edge Attributes	same as nodes	no	same as nodes	same as nodes	same as nodes
Role Attributes	no	no	no	no	yes
NACs	yes	yes	yes	yes	yes
Hyperedges	no	no	no	no	yes
Omniedges	no	see text	see text	no	yes
Rule Definition	programmed	programmed	declarative	declarative	declarative
Rule Notation	graphical	graphical	textual	textual	textual
Parameterization	all types	no	all types	graph entities	no
Rule Scheduling	story diagrams	sequence diagrams	state machine	similar regular expressions	similar regular expressions
Rule Iteration	loop	loop, recursion	loop, recursion, fix point iteration	loop, fix point iteration, transaction	loop, fix point iteration, transaction

6 Conclusion

We presented omnigraphs together with an appropriate rewrite system called *Ogre*. Omnigraphs are an extension of the well known hypergraphs, enabling the attachment of multiple nodes *and edges* to edges. In this paper, UML structures like n -ary associations and constraints between associations served as examples for the usefulness of omnigraphs. Besides model transformations, we use omnigraphs for the representation of natural language [7]. In this domain, hyper- and omniedges are essential as natural language includes complex sentence structures with higher order relations. Bond angles are an example from the domain of chemistry where one would like to declare edges between edges.

The rewrite system *Ogre* provides custom-made languages for the definition (i.e. typing) and instantiation of omnigraphs as well as the declaration of rules for their transformation. As the implementation of *Ogre* is based on a traditional graph rewrite system, we provide compilers [18] and can thus incorporate many features from our underlying system.

The reduction to normal graph rewrite systems is pretty straight forward, but the provided abstraction eases the task of specifying transformations and graphs. The automatic transformation unburdens the user from consistently and continuously regarding auxiliary nodes and edges. Instead the user can directly express his intention. This makes omnigraphs and *Ogre* *practically* more expressive than common approaches – and thus well suited to simplify model transformation for advanced UML structures.

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20. VIATRA – Visual Automated model Transformations. Dept. of Measurement and Information Systems, Budapest University of Technology and Economics.

Changes

This page outlines the changes regarding content that we have made in respect of the reviewers comments and the feedback we got on the workshop in Kassel. The entries are in no particular order.

Changes related to reviewers:

- Added a statement about the inevitable necessity of transforming the running example in a true MDD process.
- Strengthen the section on *transforming* omnigraphs. The paper did cover that topic, but it was not well structured and probably too weak.
- Clarification of our relation to “hyperedge replacement” in related work.
- Added remark about omnigraphs and bipartite graphs. In contrast to hypergraphs, omnigraphs cannot be represented by bipartite graphs.
- Added a discussion of the frequency of use of the presented UML structures and named other application areas, besides UML class diagrams and UML diagrams in general.
- Added two sections concerning the semantics of rewrite rules: one section on the effective semantics of omnigraph rewrite rules in Ogre and another section concerning the realization of these semantics on the basis of an SPO rewrite system.

Changes related to workshop feedback:

- Renamed supergraphs into omnigraphs, as some people expected our supergraphs to be related to subgraphs. Consequently renamed SUGR to Ogre.
- Weakened the accent on UML/MDD, as this was intentionally an *example* for the application of omnigraphs. By no means are omigraphs restricted to the process of MDD or representing and transforming UML.
- Included additional related work concerning AGG/ALR-graphs and GXL, including tools capable of using GXL
- Added a statement that in our approach hyperedges are not typed by the number of their tentacles, instead we allow to declare custom edge types.
- Related our role-based approach to assigning semantics via the natural order of expressions to the tentacles of hyperedges.
- Added a statement that from the viewpoint of graph theory, omnigraphs are obviously equally expressive to “traditional” graphs. But we think omnigraphs have a higher practical expressiveness.